Let S be the While-program given by

$$S \equiv (\text{ while } \neg(x = y) \text{ do } (x := x + 1; y := y - 1)\ );\ z := x$$

and let s be a state with $sx = 0$ and $sy = 4$. Compute the state s' with $(S, s) \to s'$ according to natural semantics.

We prove that $(S, s) \to s'$ with $s'x = 2$, $s'y = 2$ and $s'z = 2$. We use an abbreviation s_{ijk} to indicate a state with $s_{ijk}\ x = i$, $s_{ijk}\ y = j$, $s_{ijk}\ z = k$ and $s_{ijk}\ v = sv \quad \forall v \notin \{x, y, z\}$. We further use the following names to refer to parts of the program S:

$$\begin{aligned} S &\equiv P_1;\ P_2 \\ P_1 &\equiv \text{while } b \text{ do } S_1 \\ P_2 &\equiv z := x \end{aligned} \qquad\qquad \begin{aligned} b &\equiv \neg(x = y) \\ S_1 &\equiv S_2;\ S_3 \\ S_2 &\equiv x := x + 1 \\ S_3 &\equiv y := y - 1 \end{aligned}$$

Proof (using the inference rules of natural semantics):

$$\text{[comp]}\ \dfrac{\text{[while}^{tt}\text{]}\ \dfrac{\text{[comp]}\ \dfrac{\overset{\text{[ass]}}{\langle S_2, s_{04\_}\rangle \to s_{14\_}} \quad \overset{\text{[ass]}}{\langle S_3, s_{14\_}\rangle \to s_{13\_}}}{\langle S_1, s_{04\_}\rangle \to s_{13\_}} \quad \text{[while}^{tt}\text{]}\ \dfrac{\text{[comp]}\ \dfrac{\overset{\text{[ass]}}{\langle S_2, s_{13\_}\rangle \to s_{23\_}} \quad \overset{\text{[ass]}}{\langle S_3, s_{23\_}\rangle \to s_{22\_}}}{\langle S_1, s_{13\_}\rangle \to s_{22\_}} \quad \overset{\text{[while}^{ff}\text{]}}{\langle P_1, s_{22\_}\rangle \to s_{22\_}}}{\langle P_1, s_{13\_}\rangle \to s_{22\_}}}{\langle P_1, s_{04\_}\rangle \to s_{22\_}} \quad \overset{\text{[ass]}}{\langle P_2, s_{22\_}\rangle \to s_{222}}}{\dfrac{\langle P_1; P_2, s_{04\_}\rangle \to s_{222}}{\langle S, s_{04\_}\rangle \to s_{222}}}$$